

Serie de documentos de trabajo 14-03

Noviembre del 2014

Differentiated Social Interactions in the US
Schooling Race Gap

Luis J. Hall

Escuela de Economía, Universidad de Costa Rica

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November, 2014

Abstract

The main purpose of this paper is to study how the observed differences in educational achievement of whites and nonwhites teenagers in the US can be explained within a context of social interactions with differentiated agents, where individuals differ in how they value their interactions with individuals of their same type and individuals from the opposite type. We write a model where teenagers are allowed to interact with other teenagers and their degree of social interaction is differentiated inasmuch as whether they interact with their own group or other groups. Following an approach of differences in conditional variances, the conditions for the identification of the coefficient of differences in social interactions are established. Our estimation using the US census data on teenagers sustains the conclusion that there exist differences in the interaction coefficient between individuals of different types. Individuals will value more their interaction with individuals from their own types as opposed to individuals in the opposite type.

¹ I am grateful to my advisor Alberto Bisin for his support and advice on this project. I am also indebted particularly to Donghoon Lee, Juan A. Robalino and Pritah Dev for helpful discussions on the paper. Errors and omissions are solely due to the author. First draft, March 13, 2006.

1 Introduction

The significant disparity in the schooling achievements of different racial groups in the US has long been a focus of government and academic interest. The recent years have witnessed particular increase in public intervention in the schooling sector to reduce the achievement gap between students from different racial groups - for example, the enactment of the No Child Left Behind Act in 2002. But, the disparity still exists. The National Assessment of Educational Progress reports that since the early 1990s, the achievement gaps between White and Black and White and Hispanic have shown little measurable change (NAEP (2006)). The achievement gap is not just significantly manifest within a given school or a city, but also exhibits large variation across cities. This variation is not sufficiently explained by differences in individual and city attributes and therefore, renders standard policy remedies less effective.

In this paper, I show how this large variation across cities in the schooling racial gap (henceforth, "the gap") can be explained in a context of differentiated social interactions, where individuals value interactions with people belonging to their racial group differently from interactions with other individuals. The observation that individuals base their decisions on other people's choices, and can differentiate between individuals with different racial identity - what I call the differentiated social interaction factor, following the literature on social interactions - implies that small changes in fundamentals will have larger effects in the equilibrium outcomes. This amplifying effect, captured by the differentiated social multiplier, provides an explanation for this observed large variation. I work along the lines of Glaeser and Scheinkman (2002) to model this phenomenon.

This paper borrows from the social and cognitive psychology literature, where the idea of differentiated valuation has been extensively studied. Psychologists assert that individuals have stereotypes created, probably from their families, peers and institutions, and these stereotypes influence automatically their perception and motivation (Fiske (1998) and Worchel and Cooper (2000)). The individual will tend to overvalue those features which are self-confirming and misrepresent the others. In particular, Tajfel (1974) states along these lines that individuals will tend to value more their own group than other groups and individuals will see more homogeneity in other groups than their own groups.

In this paper I show how these differentiated perceptions of individual types can have lasting effects on the performance of different types of agents. I construct a model of social interactions in the schooling choice with differentiated agents in which social interaction means the following:

individuals' decision making is directly affected by choices made by others. While deciding on their schooling choices, teenagers take into consideration the schooling choice made by other teenagers in the city. Moreover, within a city, teenagers can distinguish teenagers who belong to their same racial group from those belonging to other groups. Accordingly, they value decisions of different types differently. That is, they exhibit *differentiated social interaction*. The theoretical model shows how the variation in the gap across cities can be explained by teenagers placing a higher value on the decisions of teenagers of their same racial group from those of the other. This distinction in the valuation translates into a social interaction multiplier of differences that has a magnifying effect on the differences across the individual types.

The assessment of whether differentiated valuation across agent types matter, nevertheless, is an empirical issue that can only be resolved with economic data. My estimation of the differentiated social interaction coefficient using the US census data supports the conclusion that there exist differentiated valuations based on racial identity. On an average, teenagers value 0.32 more years the schooling mean decisions of individuals from their same group than the schooling mean decision of teenagers from the other racial groups. Given this coefficient value, the multiplier effect is 1.47, which means that any gap in the set of exogenous factors between the two groups within the city is expanded, on an average, 1.47 times the schooling gap across the two groups.

My estimation strategy permits me to control for some potential problems which were addressed in Manski (1993) and are still present in the social interactions literature. Firstly, my strategy properly controls for the presence of unobservable heterogeneity in the data that is common to the two types of agents whites and non-whites at the city level. My variable of interest being the differences in schooling outcomes between the two types, allows me to subtract from the estimation any unobservable effect common to both types of agents in the city (see Cutler and Glaeser (1997), Card and Rothstein (2005)). Secondly, the variance approach strategy formally evades the problem of simultaneity between individuals' decisions. The approach uses the between and within variation of the data to estimate the difference in the interaction coefficients between the agent types. Hence, our approach does not require regressing individual outcomes on individuals' neighbors outcomes, and so, our strategy avoids the problem of endogeneity inherent in this regression approach (see Glaeser and Scheinkman (2002)). Thirdly, I control for location fixed effects at a higher degree of aggregation of the cities to partially reduce the degree of sorting present in the data (Scheinkman and Glaeser (2001), Graham (2004), and Cooley (2006)). Finally, the strategy follows Graham (2004) to identify the difference of the social inter-

action coefficient by means of excess variances between large and small city sizes. The variability in the size of the city permits one to control for these unobservable group-compositional effects that are common and constant across cities of different size. To facilitate initial understanding of my approach, we can think of my estimation technique as one of differences in differences estimation based on second moments or variance terms, in which, the first difference is taken between white and non-white schooling level and the second difference is taken according to the teenagers living in large or small cities.

This paper makes three additions to the existent literature. Firstly, I provide a specific model of social interaction with differentiated valuation where individuals value their own type differently from other types¹. Secondly, the paper extends Graham’s (2004) methodology for the case of two types of agents in order to identify the parameter of differentiated social interactions. Thirdly, I estimate the coefficient of differentiated social interaction using teenagers’ schooling choices and racial identity using the US census data for year 2000.

From a policy point of view, understanding the sources of the schooling gap is important. The nature of the source can lead to alternative policies with very different impacts. My study draws several policy-relevant conclusions. First, the multiplier effect on the gap implies that exogenous differences across the racial groups are amplified by the interaction process. Next, we must be careful in distinguishing between policy measures that affect the level of schooling from those affecting the gap. Measures that focus solely on increasing the schooling level can have negative effects on the size of the gap across race groups. Finally, the study supports policy measures that focus on improving the racial group with lower exogenous characteristics. These policy measures will have dual effects of increasing both levels of schooling and reducing the schooling gap.

2 Social Interactions with Differentiated Agents

Suppose there are two types of agents I and J in city c with n_c individuals of type I and m_c individuals of type J in city c . There are C cities, with $c \in C$. The sets I_{N_c} and J_{M_c} denote the sets of individuals of each type in city c so that $I_{N_c} \equiv \{I_1, \dots, I_{n_c}\}$ and $J_{M_c} \equiv \{J_1, \dots, J_{m_c}\}$ represent the n_c individuals of type I and the m_c individuals of type J in city c . Let i represent

¹Scheinkman and Glaeser (2003) provide a general framework where individuals value global and local interactions that can potentially frame this paper’s model. The model in this paper complements this idea and emphasizes the distinction in valuation of different types of agents.

any individual I -type and j any individual J -type, $i \in I_{N_c}$ and $j \in J_{M_c}$. Let a_{ci} denote the action choice of individual type I in city c for each individual $i \in I_{N_c}$ and a_{cj} the J -type individual's action in city c for each individual $j \in J_{M_c}$ in a given city c . Let t denote the individual $t \in \{I_{N_c}, J_{M_c}\}$.

Each individual t has the following Akerlof quadratic conformist indirect utility function (see Akerlof (1997), Graham (2004), Glaeser and Scheinkman (2001)) for $t \in \{I_{N_c}, J_{M_c}\}$,

$$V(a_{ct}/\lambda_c, \varepsilon_{ct}, \bar{a}_{c,T}, \bar{a}_{c,-T}) = \frac{1 - \alpha - \beta}{2} a_{ct}^2 - (\lambda_c + \varepsilon_{ct}) a_{ct} + \frac{\alpha(a_{ct} - \bar{a}_{c,T})^2}{2} + \frac{\beta(a_{c,t} - \bar{a}_{c,-T})^2}{2}$$

For this utility function, a_{ct} is the action of individual t in city c , λ_c represents some specific factor of city c and common to all individuals residing in this community. ε_{ct} refers to the taste shock for individual type t and captures heterogeneity of the individual.

The variable $\bar{a}_{c,T}$ represents the mean action of the individuals similar in type to individual t and $\bar{a}_{c,-T}$, the mean action of individuals of the opposite type of individual t . In this economy, we allow individuals to have different valuations of their interaction with members of their same type and those belonging to a different type (see Tafjel (1974), Fiske (1998) for the psychology literature on this issue, see also Scheinkman and Glaeser (2001, 2002)). The coefficients α and β capture this valuation and is common for all individuals.

The individual t chooses her action a_{ct} by maximizing her utility function so that her first order condition, the best response function, becomes,

$$a_{ct} = \lambda_c + \alpha \bar{a}_{c,T} + \beta \bar{a}_{c,-T} + \varepsilon_{ct} \quad (1)$$

Writing explicitly the first order conditions for each individual according to their type-group, the best response functions are,

$$a_{ci} = \lambda_c + \alpha \bar{a}_{cI} + \beta \bar{a}_{cJ} + \varepsilon_{ci} \quad i \in I_{N_c} \quad (2)$$

$$a_{cj} = \lambda_c + \alpha \bar{a}_{cJ} + \beta \bar{a}_{cI} + \varepsilon_{cj} \quad j \in J_{M_c} \quad (3)$$

Next we can solve for the Nash Equilibrium of this model. Since \bar{a}_{cI} and \bar{a}_{cJ} are equilibrium outcomes for each type, we calculate them explicitly in the following way. Taking the best response function for individual i in equation (2), summing over all $i \in I_{N_c}$ individuals and dividing by the total number of i -individuals n_c in c , we obtain the following equation. The case

for the individual j of J -type is identical.

$$\begin{aligned}\bar{a}_{cI} &= \lambda_c + \alpha\bar{a}_{cI} + \beta\bar{a}_{cJ} + \bar{\varepsilon}_{cI} \\ \bar{a}_{cJ} &= \lambda_c + \alpha\bar{a}_{cJ} + \beta\bar{a}_{cI} + \bar{\varepsilon}_{cJ}\end{aligned}$$

Now we can solve simultaneously for the mean action of each type in the community to obtain,

$$\begin{aligned}\bar{a}_{cI} &= \frac{\lambda_c(1 + \beta - \alpha) + \bar{\varepsilon}_{cI}(1 - \alpha) + \bar{\varepsilon}_{cJ}\beta}{(1 - \alpha)^2 - \beta^2} \\ \bar{a}_{cJ} &= \frac{\lambda_c(1 + \beta - \alpha) + \bar{\varepsilon}_{cJ}(1 - \alpha) + \bar{\varepsilon}_{cI}\beta}{(1 - \alpha)^2 - \beta^2}\end{aligned}$$

Finally using the obtained equilibrium mean actions in the best response functions, the equilibrium action for each type of individual becomes, for all $i \in I_N$ and $j \in J_M$,

$$a_{ci} = \Phi\lambda_c + \Gamma\bar{\varepsilon}_{cI} + \Psi\bar{\varepsilon}_{cJ} + \varepsilon_{ci} \quad (4)$$

$$a_{cj} = \Phi\lambda_c + \Gamma\bar{\varepsilon}_{cJ} + \Psi\bar{\varepsilon}_{cI} + \varepsilon_{cj} \quad (5)$$

with

$$\Phi = \frac{1+\beta-\alpha}{(1-\alpha)^2-\beta^2}, \quad \Gamma = \frac{\alpha(1-\alpha)+\beta^2}{(1-\alpha)^2-\beta^2}, \quad \Psi = \frac{\beta}{(1-\alpha)^2-\beta^2}$$

The equilibrium action of each individual-type is explained by three factors; common community factors, mean level tastes across type of agents, and individual heterogeneity. The first element is the common community factor that is identical for both type of individuals and in equilibrium this factor is amplified by the Φ coefficient. No differences should be observed in individual type action produced by these common factors. Next, the individual action of each type is determined by the mean composition of individual-level tastes within her type and the mean level of the other type. This average taste for each type of individual do not have the same impact on the equilibrium action of the agents. Γ is the expansion coefficient of the mean level of the individual taste in her own type. Ψ is the expansion coefficient caused by the social

interactions with the other type. An increase of one unit in the mean taste of individual-type j produces an increase of Ψ units in individuals type i and Γ units in individuals j . Depending on whether $\Gamma \geq \Psi$ or $\Gamma \leq \Psi$ one can conclude what individual type faces a larger or smaller change in her action level. The most important conclusion is that they do not have to be identical in general.

To study the differences of the equilibrium outcomes across the two type of individuals, I focus on characterizing the difference of actions between individual types. Subtracting the action of individual j from the action of individual i I obtain the following equation,

$$a_{ci} - a_{cj} = \frac{\alpha - \beta}{1 + \beta - \alpha} (\bar{\varepsilon}_{cI} - \bar{\varepsilon}_{cJ}) + (\varepsilon_{ci} - \varepsilon_{cj}) \quad (6)$$

This is an interesting result for the study of the effects of social interactions with differentiated individuals that follows below. The equation shows the relation between differences in actions of individuals that are explained by two factors, the differences in mean level across the types and differences in the individual-level heterogeneity under observation.

As expected the common factor of the community does not explain observed differences in the actions of the agents. They all face the same environment and even under a framework of social interactions these effects do not cause any deviation of one type action with respect to the other.

Next, differences in individuals' actions across types are explained by differences in the mean heterogeneity of tastes across the two types of individuals $\bar{\varepsilon}_{cI} - \bar{\varepsilon}_{cJ}$. Note however that these differences can be extended or reduced according to the degree of social interactions of individuals. A fundamental insight of this model is to show that the degree of social interaction can have an expanding or contracting effect in the observed differences in actions of the individual across types. Defining $\gamma \equiv \frac{\alpha - \beta}{1 + \beta - \alpha}$ one can study this effect. γ measures the response of the action gap across individual types to a change in the gap in mean taste. A unit increase in the difference of the mean taste across individual type translate into γ units change in the difference of actions of the individual types.

Consider the possible scenarios. If $\gamma > 0$ then any differences in mean heterogeneity across individual types is sustained in the action choice made. The gap of mean taste persists in actions. It is important to observe that a necessary condition for $\gamma > 0$ is that $\alpha > \beta$. This means that for the gap to persist individuals must value more their their type's actions than the other's.

Moreover the degree of persistence of this gap from mean taste to actions depends on whether

$\gamma > 1$, $\gamma = 1$ or $\gamma < 1$. When $\gamma = 1$ individuals interaction has no impact in affecting the size of the gap in mean taste. If $1 > \gamma > 0$ then the degree of social interaction actually reduces the difference in mean taste at the action level. A difference of one unit in mean taste translates into a difference of action by less than one. The effect is different if $\gamma > 1$. In this case, an original observed difference in mean taste is reinforced through interactions and the gap in action shows an amplified effect with respect to mean taste after the individuals choose their actions. Social interactions matter in explaining differences in actions across agents.

The realization of any of these results will depend on the particular values of the parameters α and β . I consider in this characterization only the case where $\alpha > 0$ and $\beta > 0$. Two points deserve being mentioned. First the threshold for $\gamma \geq 1$ or $\gamma \leq 1$ is $\gamma_0 = \frac{1+2\beta}{2}$. Notice that the *relative* valuation made by the individual on both types of agents directly affects the size of the gap. Moreover, and of most importance, the fact that individuals value more their own type is not sufficient to justify an amplitude effect on the gap because for γ to be larger than one, $1 + \beta > \alpha$. It is relative valuation that matters in explaining the gap in action and not only the absolute value of the coefficient of valuation of the agent's type and the other type.

To have a complete scheme of the gap observe that if $\alpha = \beta$ then the individual places the same value to the action of each type of individual and so no difference is obtained in the equilibrium outcomes due to social interactions, that is $\gamma = 0$. Finally, we can also have $0 > \gamma$ in which case differences in mean taste are reversed after action takes place. Individuals revert the gap originally observed in mean-taste. This is obtained if the above relative relation between α or β is not maintained and α grows too fast with respect to β or if $\beta > \alpha > 0$.

The following result summarizes the set of conditions characterizing their possible values of γ .

Proposition 1 *For $\alpha > 0$ and $\beta > 0$ the following relation applies;*

1. $\gamma > 0$ if and only if $1 + \beta > \alpha > \beta$.
2. $\gamma > 1$ if and only if $1 + \beta > \alpha > \frac{1+2\beta}{2}$.
3. $1 > \gamma > 0$ if and only if $\frac{1+2\beta}{2} > \alpha > \beta$.
4. $\gamma < 0$ if either $\alpha < \beta$ or $\alpha > 1 + \beta$.
5. (a) if $\alpha \rightarrow \infty$ then $\gamma = -1$ for β constant.

- (b) if $\beta \rightarrow \infty$ then $\gamma = -1$ for α constant.
- (c) if $(\alpha, \beta) \rightarrow \infty$ then $\gamma = 0$.
6. (a) If $\beta = 0$ then $\gamma = \frac{\alpha}{1-\alpha}$
- (b) If $\alpha = 0$ then $\gamma = \frac{-\beta}{1+\beta}$.

3 Identification and Estimation of Differentiated Social Interactions

The purpose of this section is to establish the set of conditions that allow for the identification of the coefficient of differences in social interaction across the individual types, $\alpha - \beta$, using my model in (6). Recall that coefficient $\frac{\alpha-\beta}{1+\beta-\alpha}$ represents the degree of social interaction of the difference across individual types that we denote by γ in equation (6). I will extend the strategy proposed by Graham (2004) for the one agent model to encompass our two differentiated agents model.

The data is composed from a sample of N individual outcomes across the C cities. For each city c , we have a vector of three observable variables previously specified that includes, first, the individual outcomes a_i and a_j for n_c individuals i and m_c individuals j respectively. Second, the size of the city denoted by $Z_c = n_c + m_c$ and finally the indicator S_c that will represent whether the city is large or small depending on the population size.

Recall that in model (6), ε_{ci} and ε_{cj} represent the individual-level heterogeneity of i and j . For instance, it can include the ability of the individual or the characteristics of the family such as parental education or income level. The terms $\bar{\varepsilon}_{cI}$ and $\bar{\varepsilon}_{cJ}$ refer to the mean of the group of i 's and j 's in c . Recall finally, that the model in differences (6) permits us to exclude the λ_c city factors affecting both i and j .

Conditioning on Z and S , the conditional means of these terms are, for all i and j ,²

$$E[(\varepsilon_{ic}, \varepsilon_{cj}) | z \in Z, s \in S] = (\mu_I(z, s), \mu_J(z, s)), \quad (7)$$

Next we introduce the following notation,

²From now on, we omit to write the city c in each variable for simplicity, i.e., μ_{Ic} instead.

$$\begin{aligned}
Var[(\varepsilon_{ic})|z \in Z, s \in S] &= \sigma_I^2(z, s) & Var[(\varepsilon_{jc})|z \in Z, s \in S] &= \sigma_J^2(z, s) \\
Var[(\varepsilon_{ic}, \varepsilon_{ic})|z \in Z, s \in S] &= \sigma_{II}(z, s) & Var[(\varepsilon_{jc}, \varepsilon_{jc})|z \in Z, s \in S] &= \sigma_{JJ}(z, s) \\
Var[(\varepsilon_{ic}, \varepsilon_{jc})|z \in Z, s \in S] &= \sigma_{IJ}(z, s) & &
\end{aligned}$$

Using this terminology, the matrix of conditional variance and covariance terms for individuals i and j becomes,

$$Var[(\vec{\varepsilon}_{ic}, \vec{\varepsilon}_{cj})|v] = \begin{pmatrix} \sigma_I^2(v) & \sigma_{II}(v) & \cdots & \sigma_{II}(v) & \sigma_{IJ}(v) & \sigma_{IJ}(v) & \cdots & \sigma_{IJ}(v) \\ \sigma_{II}(v) & \sigma_I^2(v) & \cdots & \sigma_{II}(v) & \sigma_{IJ}(v) & \sigma_{IJ}(v) & \cdots & \sigma_{IJ}(v) \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \sigma_{II}(v) & \cdots & \cdots & \sigma_I^2(v) & \sigma_{IJ}(v) & \cdots & \cdots & \sigma_{IJ}(v) \\ \sigma_{IJ}(v) & \sigma_{IJ}(v) & \cdots & \sigma_{IJ}(v) & \sigma_J^2(v) & \sigma_{JJ}(v) & \cdots & \sigma_{JJ}(v) \\ \sigma_{IJ}(v) & \sigma_{IJ}(v) & \cdots & \sigma_{IJ}(v) & \sigma_{JJ}(v) & \sigma_J^2(v) & \cdots & \sigma_{JJ}(v) \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \sigma_{IJ}(v) & \cdots & \cdots & \sigma_{IJ}(v) & \sigma_{JJ}(v) & \cdots & \cdots & \sigma_J^2(v) \end{pmatrix}$$

where we used $(z, s) \equiv (v)$ and $\vec{\varepsilon}_i$ means the vector of I individuals³. This means that individuals i and j co-vary with individuals on their own type and from individuals of the opposite type independently of their location within the group.

It is instructive at this point to study the sources of variation of the actions of our model (6). For this case, the variance of the difference in actions conditioning on the size of the city z and s denoted by v becomes, for all i and j ,

$$\begin{aligned}
Var(a_i - a_j|z \in Z, s \in S) &= (\gamma + 1)^2 \left[\frac{\sigma_I^2(v) - \sigma_{II}(v)}{n} + \frac{\sigma_J^2(v) - \sigma_{JJ}(v)}{m} + \sigma_{II}(v) + \sigma_{JJ}(v) - 2\sigma_{IJ}(v) \right] \\
&+ (n - 1) \left[\frac{\sigma_I^2(v) - \sigma_{II}(v)}{n} \right] + (m - 1) \left[\frac{\sigma_J^2(v) - \sigma_{JJ}(v)}{m} \right] \quad (8)
\end{aligned}$$

The degree of variation of our model (6) respond to the degree of variation and covariation of the individuals according to their type I and J , the city population size of each type n and m , and the interaction coefficient γ . The first element is the individual heterogeneity for each agent's type I and J denoted by σ_I^2 and σ_J^2 . This term reflects the variation in the individual heterogeneity level. Second, we have the variation produced by the composition of each individual according to their own type. In this sense, the covariance term σ_{II} represents the sources of variation obtained by the composition of individuals type I and might reflect, in particular, those elements of sorting present in the data that drove the individuals type I

³We slightly abuse notation by not distinguishing between i and I at some point of the text but the context should make clear what we are referring to in each case.

together. The covariance term σ_{JJ} has a similar interpretation for individuals type J . The last covariance term explaining the total variation is σ_{IJ} which reflects the variation obtained from the association of the two types of individuals I and J .

Notice further that the degree of social interactions expressed by γ affect the conditional variance of (8). A positive coefficient of the social interactions in the difference of actions will increase the degree of variability in actions, increase in the value of (8). If $\gamma = 0$ then the first line of formula (8) drops and the variance of the difference in action is just the sum of net variance for individuals type I and J independently weighted by the size of the population of each group. In summary and as argued in the introduction, social interactions directly affect the degree of variability observed in the model and impose strong conditions of separability of the terms in formula (8). We need a further assumption to disentangle the effect of social interaction and identify γ .

We focus next in the relationship of the degree of variance between cities and within cities conditioning on the city-size to establish the conditions for identification of our coefficient. Given the one error type form of (8) we can introduce the following transformation of the the set of actions a_i and a_j in the spirit of between and within variances.

With this purpose in mind, we make the following transformation to our data defining first the between cities action transformation as,

$$h_c^b \equiv (\bar{a}_I - \bar{a}_J - [\mu_I(s) - \mu_J(s)])^2, \quad (9)$$

where $\mu_I(s)$ and $\mu_J(s)$ were previously defined and are now only conditioned on $S = s$ and averaged over Z . The following transformation is,

$$h_c^w \equiv \sum_{i \in I_N} \sum_{j \in J_M} \left[\frac{(a_i - \bar{a}_I)}{\sqrt{n-1}} - \frac{(a_j - \bar{a}_J)}{\sqrt{m-1}} \right]^2 \quad (10)$$

Conditioning on $s \in S$ and averaging over the the city-size Z we take the conditional expectations of (9) and (10). The first expression is for the conditional expectation of the transformation (9) that becomes,

$$E[h_c^b | s \in S] = (\gamma + 1)^2 \left[\frac{\sigma_I^2(s) - \sigma_{II}(s)}{n} + \frac{\sigma_J^2(s) - \sigma_{JJ}(s)}{m} + \sigma_{II}(s) + \sigma_{JJ}(s) - 2\sigma_{IJ}(s) \right]. \quad (11)$$

The second conditional expectation for the formula (10) becomes,

$$E[h_c^w | s \in S] = \left[\frac{\sigma_I^2(s) - \sigma_{II}(s)}{n} + \frac{\sigma_J^2(s) - \sigma_{JJ}(s)}{m} \right]. \quad (12)$$

The between variance expression (11) represents how much variation is observed in the difference of action across the cities while expression (12) represents the degree of variation of actions in differences within the city. Notice that the social interaction term γ is only present in the between variance expression (11). Hence, under the presence of social interactions, the between variance expression will be expanded due to the larger differences observed across the cities. The within variance component is not dependent of the social interaction term.

An important result due to Graham (2004) states that the conditional expected value of the within term is one of the components of the conditional expectation of the between term in the case of the one agent model. Our results (11) and (12) proves that this condition can also be found in our model of two agents with differentiated social interaction valuations. The equation (11) can then be expressed in terms of (12) as follows,

$$E[h_c^b | s \in S] = (\gamma + 1)^2 [E[h_c^w | s \in S] + \sigma_{II}(s) + \sigma_{JJ}(s) - 2\sigma_{IJ}(s)].$$

If we call

$$\Theta(s) \equiv \sigma_{II}(s) + \sigma_{JJ}(s) - 2\sigma_{IJ}(s),$$

then we can write the conditional expectation as,

$$E[h_c^b | s \in S] = (\gamma + 1)^2 [E[h_c^w | s \in S] + \Theta(s)], \quad (13)$$

and we will have the following conditional moment to test for the social interaction coefficient,

$$E[h_c^b - (\gamma + 1)^2(h_c^w - \Theta(s)) | s \in S] = 0. \quad (14)$$

In order to identify this model we assume that the following covariance elements are constant across the city-size,

$$\sigma_{II}(s) = \sigma_{II}, \quad \sigma_{JJ}(s) = \sigma_{JJ}, \quad \sigma_{IJ}(s) = \sigma_{IJ}, \quad (15)$$

and that the variance inside the groups (12) differs across S . In formal terms, we require that

$$E[h_c^w | s \in S] \neq E[h_c^w | s' \in S], \quad \text{for } s \neq s', \quad \text{and } s, s' \in S \quad (16)$$

Using conditions 15 on the conditional moment equation (14) for s and s' in S and subtracting both equations, the constant terms are cancelled out under assumption (15). Next, under assumption (15) we can solve for $(1 + \gamma)^2$ to obtain,⁴

$$(1 + \gamma)^2 = \frac{E[h_c^b | s \in S] - E[h_c^b | s' \in S]}{E[h_c^w | s \in S] - E[h_c^w | s' \in S]} \quad (17)$$

There are several methods applicable to estimate $(1 + \gamma)^2$. One strategy proposed by Graham (2004) is to use a Wald test. We first estimate $(\mu_I(s) - \mu_J(s))$, next we calculate the values of (9) and (10) to obtain h_b and h_w , and finally we regress h_b on h_w instrumenting h_w on S where S is an indicator variable, in our case, large and small cities. The standard error are correct though the method is not asymptotically efficient. This strategy will be followed below in our application.

4 Differentiated Social Interactions in the US Schooling Race Gap

4.1 Characterization of the data

The data employed is from the US 2000 Census containing 5% of the US population. For the purpose of this study we select teenagers at the age of 18 years old as our unit of analysis. The sample contains 201,868 teenagers at this age. These teenagers have reached an age where

⁴This proof of identification is provided by Graham (2006) in Proposition 1.1. of his paper

they potentially can have completed their high school education. We are especially interested in studying the degree of differentiated social interactions in schooling decisions during high school years.

According to the model proposed before, individuals make their decisions based on their own characteristics and their peers' decisions differentiating on the type of the individual. We take the numbers of years of education completed at the moment of the interview as our variable of choice, a . Individuals report the grade of school completed and we transform this grade into a number representing the years of education.⁵ On average the number of years completed is 11.40 years of education with 1.49 standard deviation.

The unit of interaction in this paper is the city. The set of city locations C employed is composed by the Public Use Micro Data (PUMA) category that divides the US territory in 2071 PUMA units, each one with population of at least 100.000 individuals. The PUMA unit is an ideal location scenario since it is obtained from grouping mostly connected core metropolitan statistical areas. The general concept of a metropolitan or micropolitan statistical area is that of a core area containing a substantial population nucleus, together with adjacent communities having a high degree of economic and social integration with that core. We will refer to this PUMA unit as the city. Moreover, it is important to add that PUMA units are fully contained into SUPERPUMA units which are cities with population above 400.000 people. There are 532 SUPERPUMA and each of them is fully contained within an State. This distinction between PUMA, SUPERPUMA and State will be exploited later on for controlling our results and reduce the sources of heterogeneity.

Individuals report their racial group in the Census. We group the individuals according to their race by white and nonwhites and we refer to i individuals as white teenagers and j individuals as non-white teenagers with a_i and a_j representing their school achievement decision

⁵Several comments on this assignment. 1) Grades 4 and 5 are reported as one in the interview and we take the average number for this group; 0.7% of the sample have teenagers with this schooling achievement. Grade 7 and 8 also present the same potential problem but the information about completing 7 or 8 can be retrieved from combining this question with the question on school attendance. However, for 2,362 students this is not possible since they are not attending school so that we used the average instead. 2) teenagers can repeat a grade. This is not a problem because we do not consider repetition of a year as another year of education. 3) Differentiation across states in the meaning of 'grade'. This problem is potentially more difficult but for that we also control later at State and Super-puma level.

respectively.⁶⁷ With this grouping, I am treating all non-white individuals, including particularly, Blacks, Hispanics and Asians, as one group. I have followed this grouping to simplify the calculation below. The methodology could be extended for more groups and this will impose longer calculations. Focusing only in some particular pair of racial groups, say Whites and Blacks, will imply that some other groups that co-exist in the same city will be ignored in the estimation. This might seriously affect the estimation as the interaction of these groups with the included groups might be very important. My results below show that there exist a differentiated valuation across agents under this non-white grouping. Excluding, for instance, Asian teenagers from the non-white group will accentuate the schooling gap and strength the results⁸

According to race, the sample is divided between 71,505 non-whites teenagers and 130,353 whites with an average educational achievement of 11.17 years of education for non white teenagers and 11.53 years of education for whites. There is an average gap on schooling achievement of 0.36 years between white and nonwhites teenagers at the age of 18 in the US. Table 1 in the appendix presents the schooling achievement according to race.

As argued in the introduction of this paper, there is considerable variation of this schooling gap across cities. Positive values of the gap means higher schooling for white teenagers. The gap of schooling ranges from cities where whites have 2 fewer years of schooling than nonwhites to cities where whites have about 3 years more of education.⁹ Figure 1 and Figure 2 show the schooling gap across cities in more detail. Figure 1 is the histogram of the schooling gap. The distribution is negatively skewed and centered at the 0.36 gap mean. Figure 2 reinforces this point. The top of the figure shows the means of schooling for white and non-white teenagers -with a solid line for the white students- and the bottom of the figure shows the schooling gap per city. Notice again the high concentration of the gap on the positive portion of the figure. On average the mean and standard deviation of the positive gap is 0.64 and 0.55 respectively in

⁶The US 2000 Census introduced the possibility that individuals answer their race to various groups. This might affect our distinction between whites and non-whites. However, the number of people reporting two category in this category is very low

⁷To be more precise about the concept of race, the US Census Bureau establishes that 'The concept of race, as used by the Census Bureau, reflects self-identification by people according to the race or races with which they most closely identify. These categories are socio-political constructs and should not be interpreted as being scientific or anthropological in nature. Furthermore, the race categories include both racial and national-origin groups.' page 337, US Census Bureau (2003).

⁸Moreover, as a robustness check, I have estimated the model under different grouping and the results below hold.

⁹3 cities present larger negative values one of -5 and two above -10 but these are extreme values

comparison to -0.35 and 0.38 for the cities with negative gap as shown on Table 1.

Besides the variation in the schooling gap across cities, there is also high variation in the population size of the city, for our purpose, the number of teenagers in the city. This sort of variation is a requirement for the applications of the methods proposed in the previous section for estimating the coefficient of differences in social interactions. The size of the teenager population across these cities ranges from 12 to 323 teenagers. Figure 3 shows the distribution of the size of the population per city. The distribution is skewed to the right and centered at 97.6 individuals.

One of the requirements for the correct application of the strategy proposed in the previous section is that the variance in the schooling gap across cities should vary with the city size. In a nutshell, the identification condition previously established asserts that some of the covariances components of the agent-types, in particular the covariance of whites-whites, nonwhites-nonwhites and whites-nonwhites are constant across large and small cities so that those sources of variation can be subtracted off across city-size from the between variance. Casual observation of our data permits to study this critical point of variation of the conditional variances. In particular, consider Figure 4 where we are presenting the schooling gap in terms of the city size. Notice the strong variability of the schooling gap for cities with lower population.

The degree of dispersion of the city schooling gap for cities with lower population is larger than those cities highly populated. For the purpose of this application in this paper, we have employed 116 teenagers as the cut off population size. We define large cities as those cities with more than 116 individuals and small cities those cities with less than 116 individuals. This number divides the number of cities into small and large cities with similar population in each group of cities¹⁰.

Using this cut off point, the standard deviation of cities with more than 116 teenagers is 0.62 and lower than the 0.69 standard deviation of cities with less than 116 teenagers as shown on Table 2. As observed before and shown on this Table, the degree of variability is higher in those cities with positive gap than on those cities with negative gap.

Table ?? presents my results for the formal test for difference of conditional variances. My test is statistically meaningful at 99% and shows that at the threshold of 116 teenagers, the small cities present larger variability in the size of the gap across cities than for those cities with large population.¹¹ In particular, the variance in the schooling gap conditioning on small cities

¹⁰I also considered the cut off point of 90 individuals. This point produces the same quantity of cities in each group instead of dividing the population in half and half. The results follow in the same direction though the effects are smaller.

¹¹See the footnote of the Table for particular details about the testing of differences in the conditional between

is 0.29 compared to 0.23 in large cities.¹²

4.2 Coefficient of Differences in Social Interactions

My goal is to apply the test proposed in (17) to estimate the coefficient $(1 + \gamma)^2$. In order to undertake this test, I must apply first formulas 9 and 10 to the data. To exert a better control for the dispersion of the data across states and cities I introduced dummy control variables at the level of the super-city, referred before as SUPERPUMA. Recall that cities are contained in super-cities. As proposed and applied by Scheinkman and Glaeser (2001), Graham (2004) and Cooley (2006), introducing this set of supercity-dummy variables will fully control for the degree of sorting across super-cities and leave only unexplained this effect within super-cities¹³.

To estimate formula 9, we need to obtain $\mu_i(s) - \mu_j(s)$. I first control for the means at the SUPERPUMA level and then calculate the means across the city class, that is, large and small cities. This set of transformation in the data will not invalidate the standard errors (see Graham (2004)).

Table 4 presents the results of the estimation of $(1 + \gamma)^2$. In the first row, I have estimated the coefficient by applying proposition 2 and obtaining the differences of the between conditional variance divided by the differences of the 'expected' conditional variance. My estimate is 2.346 and meaningful at the 99% confidence.¹⁴ The coefficient is positive, large and meaningful implying that there are differences in the individual valuation of the two different type of individuals i and j . Recall that $\gamma = \frac{\alpha - \beta}{1 + \beta - \alpha}$ so that for these values we obtain $\alpha - \beta \simeq 0.34$. These values imply that individuals value differently their interactions with their own individual-types from those they consider from the other type. In particular, the difference in valuations is on the magnitude of 0.34 so that their own type is more valued than the opposite type.

Notice that my estimation permits to assess the differences in the interaction coefficients but not the level of any of the coefficients. Our strategy is silent to this value¹⁵. Moreover, it is inappropriate to compare previous estimation of the coefficient of social interaction obtained

variance.

¹²Another relevant issue is sampling error caused by the fact that the sample size is the same as the city size in this case. I further restricted all cities in my sample to the same sample size and found out that the differences in variances are still statistically meaningful which is the requirement for our test.

¹³In this sense, I work with residuals of the regression on this set of super-city dummies.

¹⁴I study the positive root of this value as it is the one suggested from the model.

¹⁵Notice that one must be careful in extending this estimation to the level of the coefficient. Such estimation would require strong assumptions about the behavior of the unobservable variables at the level of the city and affecting both individuals.

from other studies and apply these values into this model. Those models function under the assumption that individuals value equally their interaction with all agents, a major point of distinction from the model proposed in this paper.

On the second row of Table 4, I extend the previous estimation in the following sense. Recall from the estimation section, that one approach to estimate the model is to obtain the h_b and h_w values for each city and then regress h_b on h_w instrumenting for the class of city, large or small. On the second row we follow this strategy but include the set of controls of super-cities previously explained into this regression to exert a better control for the variability across super-cities. The value of the coefficient is reduced to 1.531 and almost meaningful at the 90% confidence. This value implies that the coefficient of differences $\alpha - \beta$ becomes 0.19. The difference is still large and meaningful and our previous interpretation applies.

The last two lines follow a similar approach proposed before but now we control for State instead of super-cities. Recall that super-cities are fully contained in the state. The reduction in the coefficient is due to the application of a less precise control variable. Recall that my estimation strategy first focuses on obtaining an estimated value of $\mu_i(s) - \mu_j(s)$ in order to apply formulas 9 and 10. The application of a less precise variable means that the difference on these coefficients will actually increase and reduce the between variance values and also the difference in the values conditioning on the size of the city.¹⁶

Under the identification assumptions proposed, in particular constant covariance terms between white-white, nonwhite-nonwhite, and white-nonwhite across large and small cities, I can conclude that individuals types, whites and nonwhites have different valuations of their social interaction with individuals of their same type and those from the opposite type, the difference in the coefficient terms is in the range of 0.20 and 0.32 and meaningful.

5 Conclusion

The main purpose of this paper is to study how the large observed differences in the schooling achievement of white and nonwhite teenagers across cities in the US can be explained in a context of social interactions with differentiated agents, in which individuals have a different value of their interactions with individuals belonging to their same racial group versus individuals belonging

¹⁶A final robustness check focuses on controlling for the sampling error as discussed in footnote 11. I have repeated the first experiment reducing the sample size of all cities to an uniform sample size of 60 for all cities above this size. As said before, there is still a significative difference in the variance for large and small cities. The coefficient is reduced in this case. Further testing must be carried out in this direction.

to the other racial groups. In order to assess this problem, the paper proposed a model of social interaction with differentiated agents. The conditions for the identification of the coefficient of differentiated social interactions were established.

My estimation using the US census data on teenagers sustains the conclusion that there exist differences in the interaction coefficient between individuals of different types in the size of 0.32. This produces a multiplier effect of 1.47 which implies that any original differences across the groups will be amplified in this amount. Another way to read this result is to say that the differentiated social interaction behavior explains almost one third of the schooling gap across whites and non-whites in the US.

The US census data has important advantages. It is one important source of data with large coverage across space and racial groups in the US. Moreover, it is open to many sources of variability. Other more experimental sources of data can enhance our results while providing further control on other sources of variation. The methodology proposed is general and can be applied to other type of problems in other fields.

Further research is required in the formation of the groups and the implications in the social interaction pattern. Some assumptions on the sources of variation across groups in our model can be relaxed to enrich our comprehension of the social interaction effect. In particular, we can introduce city-factors that are uncommon to the racial groups in the city.

Another interesting line of research focuses on the racial composition of the cities and the effects on the social interaction. In particular, it will be interesting to better understand segregation patterns in the data and its relationship to the differentiated social interaction factor.

6 Tables and Figures

Table 1: Schooling Achievement of 18 years old Teenagers

Schooling Achievement Number of Years	Small Cities (Population < 117)			Large Cities (Population > 117)			All Cities			
	Small Cities		Total	Large Cities		Total	Small City		Large City	Grand Total
	Non-Whites	Whites		Non-Whites	Whites		Small City	Large City		
0	457	227	684	205	116	321	662	343	1,005	
4	169	117	286	29	8	37	198	125	323	
5.5	811	477	1,288	166	75	241	977	552	1,529	
7	121	66	187	115	64	179	236	130	366	
7.5	802	485	1,287	557	506	1,063	1,359	991	2,350	
8	202	101	303	152	101	253	354	202	556	
9	2,163	1,261	3,424	1,641	1,086	2,727	3,804	2,347	6,151	
10	3,930	2,139	6,069	4,803	3,335	8,138	8,733	5,474	14,207	
11	12,680	6,838	19,518	29,410	20,707	50,117	42,090	27,545	69,635	
12	17,392	9,676	27,068	24,232	16,830	41,062	41,624	26,506	68,130	
13	6,163	4,323	10,486	12,969	12,806	25,775	19,132	17,129	36,261	
14	79	32	111	108	53	161	187	85	272	
16	1		1				1		1	
Total	44,970	25,742	70,712	74,387	55,687	130,074	119,357	81,429	200,786	

Note: Fifteen cities are excluded from the sample since 5 contains only non-white individuals and 10 only whites. The total number of observations excluded is 1072. I also drop 2 cities with extreme deviation between whites and nonwhites. Figures 1 to 4 are based on these 2054 cities.

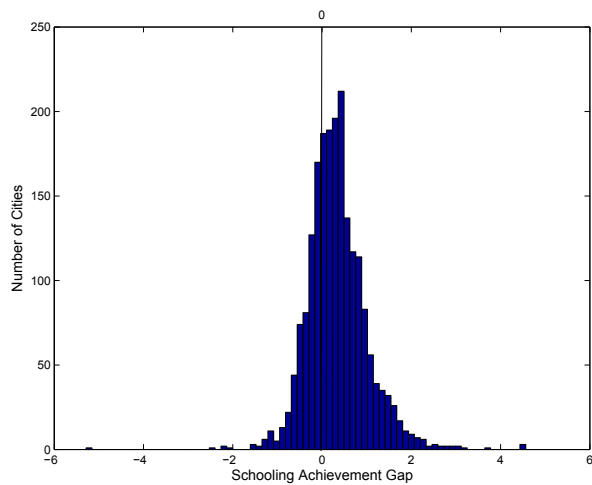


Figure 1: US Schooling Gap Histogram

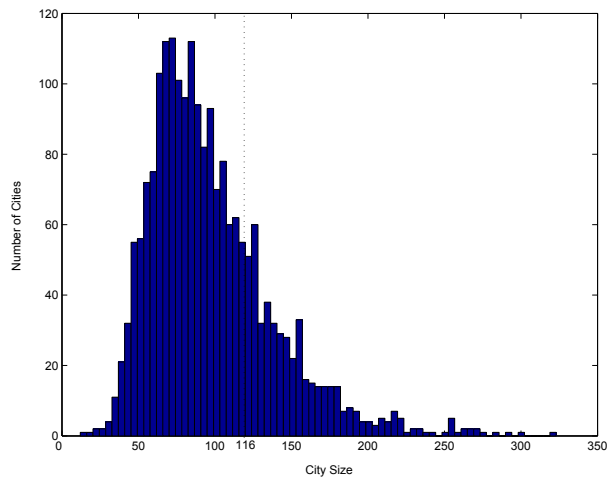


Figure 3: US City Population Size Histogram

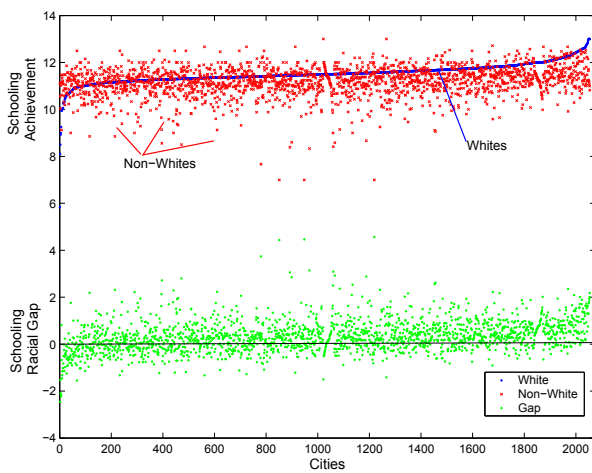


Figure 2: US Race Schooling Achievement and Schooling Gap per City

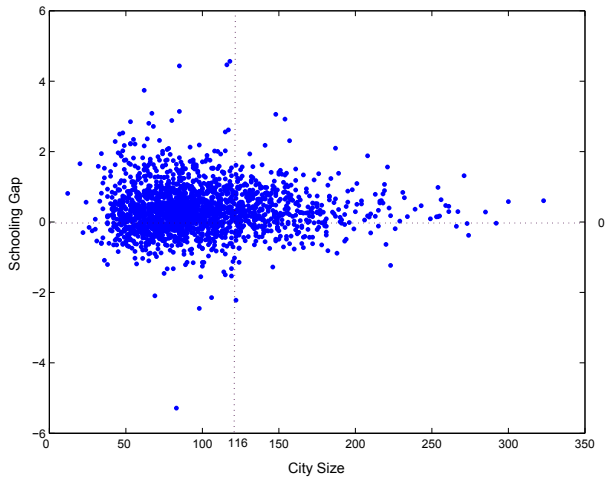


Figure 4: US Schooling Gap by City Population Size

Table 2: The City Schooling Gap according to the City Population Size

Schooling Gap	Small Cities (Population < 117)					Large Cities (Population > 117)					All Cities	
	0-60	61-90	91-116	Total	117-180	181-240	241 +	Total	Grand Total			
Negative Gap												
Mean	-0.34	-0.33	-0.40	-0.35	-0.34	-0.36	-0.14	-0.34	-0.35	-0.35		
St. Dev.	0.27	0.42	0.42	0.39	0.35	0.35	0.16	0.34	0.38	0.38		
Percentage White	0.68	0.66	0.71	0.68	0.73	0.81	0.88	0.75	0.69	0.69		
Number Cities	98	259	126	483	118	18	4	140	623	623		
Number Teenagers	4,801	19,158	13,013	36,972	16,585	3,533	1,105	21,223	58,195	58,195		
Positive Gap												
Mean	0.70	0.65	0.62	0.65	0.61	0.58	0.46	0.60	0.64	0.64		
St. Dev.	0.60	0.57	0.52	0.56	0.52	0.47	0.33	0.50	0.55	0.55		
Percentage White	0.66	0.59	0.59	0.60	0.64	0.69	0.79	0.65	0.62	0.62		
Number Cities	201	485	350	1,036	329	51	15	395	1,431	1,431		
Number Teenagers	10,026	36,612	35,596	82,234	45,851	10,364	3,991	60,206	142,440	142,440		
Total Gap												
Mean	0.36	0.31	0.35	0.33	0.36	0.33	0.33	0.36	0.34	0.34		
St. Dev.	0.71	0.70	0.67	0.69	0.64	0.60	0.39	0.62	0.68	0.68		
Percentage White	0.66	0.62	0.62	0.63	0.66	0.72	0.81	0.67	0.64	0.64		
Number Cities	299	744	476	1,519	447	69	19	535	2,054	2,054		
Number Teenagers	14,827	55,770	48,609	119,206	62,436	13,897	5,096	81,429	200,635	200,635		

Note: Fifteen cities are excluded from the sample since 5 contains only non-white individuals and 10 only whites. The total number of observations excluded is 1072. I also drop 2 cities with extreme deviation between whites and nonwhites. Figures 1 to 4 are based on these 2054 cities.

Table 3: Conditional Variance on City Population Size Test

Large Cities	0.2362 (0.0220)
Small Cities	0.2922 (0.0131)
Differences	0.0560 (0.0255)
<hr/>	
Number of Cities	2010
Number of Large Cities	1480
Number of Small Cities	530

Note: Fifteen cities are excluded from the sample since

7 Bibliography

References

- [1] Akerlof, George A. (1997): "Social distance and social decisions," *Econometrica* 65 (5), 1005-1027.
- [2] Becker, G. (1997): "Social Economics," University of Chicago mimeograph.
- [3] Brock, W. and S. Durlauf (1995): "Discrete Choice with Social Interactions I: Theory" NBER Working Paper # 2591.
- [4] Card, and Jesse Rothstein (2005): "Racial Segregation and the White-Black Score Gap," NBER Working Papers 12078, National Bureau of Economic Research.
- [5] Case, A. and L. Katz (1991): "The Company You Keep: The Effect of Family and Neighborhood on Disadvantaged Youth," NBER Working Paper No. 3708.
- [6] Cooley, Jane (2006): "Disaggregation and the Achievement Gap: Do Diverse Peers Help?," Duke University.

Table 4: Differentiated Agent Social Multiplier GMM Estimates

	Multiplier Coefficient $(1 + \gamma)^2$
No Control Variables	2.346 (0.916)
Control Variables: Super City (532)	1.531 (0.942)
No Control Variables	1.616 (0.807)
Control Variables: State (51)	1.243 (0.749)
City Size cut off point	116
Sample Size	2010

- [7] Cutler, David M. and Edward L. Glaeser (1997): "Are Ghettos Good or Bad?" *Quarterly Journal of Economics* 112, 827-72.
- [8] Fiske, S. T. (1998): "Stereotyping, Prejudice, and Discrimination," In D. T Gilbert, S. T. Fiske and G. Lindzey (eds) *Handbook of Social Psychology* (4th edition, Vol 2., pp. 357-411). New York, McGraw-Hill.
- [9] Fryer, Roland G. and Matthew O. Jackson (2003): "Categorical Cognition: A Psychological Model of Categories and Identification in Decision Making," NBER Working Papers 9579, National Bureau of Economic Research.
- [10] Glaeser, E., Bruce Sacerdote and José A. Scheinkman (1996): "Crime and Social Interactions," *The Quarterly Journal of Economics* 111(2), 506-548.
- [11] Glaeser, E. and José A. Scheinkman (2001): "Measuring Social Interactions," in *Social Dynamics* edited by S. Durlauf and P. Young. Cambridge: MIT Press.
- [12] Glaeser, E. and José A. Scheinkman (2002): "Non-Market Interactions," published in *Advances in Economics and Econometrics: Theory and Applications, Eight World Congress*, M. Dewatripont, L.P. Hansen, and S. Turnovsky (eds.), Cambridge University Press, 2002.
- [13] Graham, Bryan S. (2004): "Identifying Social Interactions through Excess Variance Contrasts," Working Paper, Berkeley University.
- [14] Jones, Stephen R. G. (1990): Worker Interdependence and Output: The Hawthorn Studies Reevaluated, *American Sociological Review* 55, 176-190.
- [15] Manski, C. (1993): Identification of Endogenous Social Effects: The Reflection Problem, *Review of Economic Studies*, 60, 531-542.
- [16] U.S. Department of Education, National Center for Education Statistics. (2006). *The Condition of Education 2006, "Trends in the Achievement Gaps in Reading and Mathematics"*, NCES 2006-071, Washington, DC: U.S. Government Printing Office.
- [17] Schelling, T. (1978): "Micromotives and Macrobehavior." New York: Norton.
- [18] Tajfel (1974): "Social Identity and Intergroup Behavior," *Social Science Information/Sur les Sciences Sociales*. 13, 65-93.

- [19] US Census Bureau, (2003): Technical Documentation: Census 2000, Public Use Microdata Sample, (PUMS), United States, Technical Documentation.